AD-A100 868 R AND D ASSOCIATES MARINA DEL REY CA F/G 20/14
EXTENDED THEORY FOR APPROXIMATE ELF PROPAGATION CONSTANTS IN TH--ETC(U)
SEP 79 C GREIFINGER, P GREIFINGER DNA001-79-C-0011
UNCLASSIFIED RDA-TR-110005-001 DNA-5561T NL

1 OF 1
AD A
100868

END
DATE
FILMED
7-81
DTIC

DNA 5561T

AD A100368

EXTENDED THEORY FOR APPROXIMATE ELF PROPAGATION CONSTANTS IN THE EARTH-IONOSPHERE WAVEGUIDE

Carl Greifinger
Phyllis Greifinger
R&D Associates
P.O. Box 9695
Marina del Rey, California 90291

1 September 1979

Topical Report for Period 27 November 1978—1 September 1979

CONTRACT No. DNA 001-79-C-0011

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.

THIS WORK SPONSORED BY THE DEFENSE NUCLEAR AGENCY UNDER RDT&E RMSS CODE B310079464 P99QAXDB00144 H2590D

Prepared for
Director
DEFENSE NUCLEAR AGENCY
Washington, D. C. 20305

Destroy this report when it is no longer needed. Do not return to sender.

PLEASE NOTIFY THE DEFENSE NUCLEAR AGENCY, ATTN: STTI, WASHINGTON, D.C. 20305, IF YOUR ADDRESS IS INCORRECT, IF YOU WISH TO BE DELETED FROM THE DISTRIBUTION LIST, OR IF THE ADDRESSEE IS NO LONGER EMPLOYED BY YOUR ORGANIZATION.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: DNA 5561T
2. GOVT ACCESSION NO: AD-A100868
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): EXTENDED THEORY FOR APPROXIMATE ELF PROPAGATION CONSTANTS IN THE EARTH-IONOSPHERE WAVEGUIDE.
5. TYPE OF REPORT & PERIOD COVERED: Topical Report for Period 27 Nov 78—1 Sep 79
6. PERFORMING ORG. REPORT NUMBER: RDA-TR-110005-001
7. AUTHOR(s): Carl Greifinger, Phyllis Greifinger
8. CONTRACT OR GRANT NUMBER(s): DNA 001-79-C-0011
9. PERFORMING ORGANIZATION NAME AND ADDRESS: R & D Associates, P.O. Box 9695, Marina del Rey, California 90291
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: Subtask P99QAXDB001-44
11. CONTROLLING OFFICE NAME AND ADDRESS: Director, Defense Nuclear Agency, Washington, D.C. 20305
12. REPORT DATE: 1 September 1979
13. NUMBER OF PAGES: 34
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):
15. SECURITY CLASS. (of this report): UNCLASSIFIED
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE: N/A

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

This work sponsored by the Defense Nuclear Agency under RDT&E RMSS Code B310079464 P99QAXDB00144 H2590D.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

ELF Propagation, Earth-Ionosphere Waveguide, Propagation Parameters, Phase Speed, Attenuation Rate, Excitation Factor.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Simple analytic approximate expressions have been derived for ELF propagation constants in the earth-ionosphere waveguide. The expressions are an extension of previous work to a more general class of ionospheric-conductivity height profiles. The expressions derived here allow the rapid computation of ELF phase speeds, attenuation rates, and excitation factors for a wide range of ionosphere conditions without the necessity of lengthy full-wave computer calculations. The results can be applied to the rapid

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

(12) 37

410124

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

20. ABSTRACT (Continued)

evaluation of the effects of a variety of ionospheric disturbances, both natural and artificial, on ELF communication systems.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

TABLE OF CONTENTS

A

I. INTRODUCTION

In two recent papers (Greifinger and Greifinger [1978] and Greifinger and Greifinger [1979], hereinafter referred to as Paper I and Paper II), the authors derived simple analytic approximate expressions for TEM eigenvalues for ELF propagation in the earth-ionosphere waveguide, assuming that the ionospheric conductivity profile could be approximated in a certain convenient way. These expressions make possible a rapid calculation of ELF phase velocities and attenuation rates without the necessity of lengthy full-wave numerical calculations. Comparisons were made, for a number of ionospheric-conductivity profiles, between approximate and full-wave eigenvalues, and the results were found to be in excellent agreement in all cases. More recently, Bannister [1979] has used our approximate theory and the Wait [1964, 1970] VLF exponential ionospheric-conductivity profile to determine propagation constants for ELF daytime propagation in the earth-ionosphere waveguide. He showed that the resulting values of ELF attenuation rate and phase velocity are in excellent agreement with measured data from various sources in the frequency range from 5 to 2000 Hz.

The work of Bannister clearly demonstrates the validity and calculational utility of the approximate theory. It also adds credence to the belief that the theory should be equally applicable to PCA-disturbed ionospheres, for which comparable data do not yet exist, and to nuclear-disturbed ionospheres. As such, it is potentially very useful for making rapid and inexpensive sensitivity studies of ionospheric propagation under disturbed conditions. It therefore seems useful to extend the approximate theory to a more general class of ionospheric-conductivity profiles than those considered in Papers I and II. The conductivity profiles to which the

PRECEDING PAGE BLANK-NOT FILMED

results in Papers I and II apply will appear as limiting cases of the more general profile. In addition, approximate analytic expressions will be derived, within the framework of the theory, for the excitation factor, which is an important additional measurable quantity in ELF propagation experiments.

II. SUMMARY OF PREVIOUS WORK

In Paper I, the authors developed the theory for an ionosphere that was sufficiently disturbed that there was no significant penetration of the electromagnetic field to altitudes where anisotropy due to the Earth's magnetic field had to be taken into account. It was shown that the propagation constant, under these conditions, was determined by four parameters – two frequency-dependent altitudes and the local conductivity scale height at each of those altitudes. The lower altitude, denoted by h_o, is the altitude at which the conduction current becomes equal to the displacement current. The higher altitude, denoted by h_1, is where the reciprocal of the local wave number becomes equal to the local scale height of the refractive index. If the conductivity profile in the neighborhood of h_o and h_1 can be adequately approximated by a local exponential, the expression for the TM eigenvalue is

$$S_o^2 = \frac{\left(h_1 + \frac{i\pi}{2}\zeta_1\right)}{\left(h_0 - \frac{i\pi}{2}\zeta_0\right)} \tag{1}$$

where ζ_0 and ζ_1 are the local scale heights at h_0 and h_1, respectively. From the definition of S_o, the relative phase velocity is

$$\frac{v}{c} = R_e(S_o) \approx \left(\frac{h_0}{h_1}\right)^{1/2} \tag{2}$$

and the horizontal attenuation rate in decibels per megameter is

$$\alpha = 8.68 \times 10^{3} \frac{\omega}{c} \operatorname{Im}(S_o) \approx 0.143\, f \left(\frac{h_1}{h_0}\right)^{\frac{1}{2}} \left(\frac{\zeta_0}{h_0} + \frac{\zeta_1}{h_1}\right) \qquad (3)$$

where f is the frequency in Hertz.

In Paper II, the theory was extended to include the effects of anisotropy. For the sake of mathematical simplicity, it was assumed that the Earth's field was vertical, but the results actually apply over a rather large geographic range. When anisotropy is included, the parameters which determine the propagation constant differ for daytime and nighttime ionospheric conditions. For both sets of conditions, two parameters which enter are the frequency-dependent altitude h_0 at which the conduction current parallel to the magnetic field becomes equal to the displacement current, and the local scale height of the parallel conductivity ζ_o. Under daytime conditions, two additional pairs of frequency-dependent altitudes and associated scale heights appear as parameters. One altitude is that at which the reciprocal of the local wave number for vertically propagating O waves becomes equal to the local scale height of the refractive index for such waves. The associated scale height is the scale height of this refractive index. The other pair of parameters are the corresponding quantities for vertically propagating X (whistler) waves. If these altitudes are attained in a region of the ionosphere wave $|\sigma_H| >> |\sigma_P|$, where σ_H is the Hall conductivity and σ_P the Pedersen conductivity, the two pairs of parameters become identical. This is, in fact, the case over a substantial altitude range. Under these conditions, the approximate expression for the TEM eigenvalue is

$$S_o^2 = \frac{h_1(h_1 + i\pi\zeta_1)}{\left(h_0 - \frac{i\pi}{2}\zeta_0\right)\left(h_1 + \frac{i\pi}{2}\zeta_1\right)} \tag{4}$$

where the altitude h_1 and scale height ζ_1 are referred to the Hall conductivity. This expression is slightly different from Eq. (1), its counterpart for the isotropic ionosphere. However, since $\zeta_0/h_0 << 1$ and $\zeta_1/h_1 << 1$, the value of S_0 is the same to lowest order in these quantities.

For typical nighttime conditions, a sharp reflecting E-region bottom may be encountered before the reciprocal of the local wave number becomes equal to the local scale height. Under such circumstances, the altitude of the E-region bottom replaces h_1 as a parameter, and the local wavelength on the E-region side of the bottom replaces the scale height as a parameter. Under these conditions, the approximate TEM eigenvalue is given by

$$S_o^2 = \frac{h_E}{\left(h_0 - \frac{i\pi}{2}\zeta_0\right)}\left\{1 + \frac{\varepsilon}{2}\left[\frac{1 + i(1 + 2\varepsilon)}{1 + \frac{i}{2}\varepsilon(1 + i)}\right]\right\} \tag{5}$$

where h_E is the height of the E-region bottom and

$$\varepsilon = \frac{1}{k_0 n_E h_E} \tag{6}$$

where $k_0 = \omega/c$ and n_E is the index of refraction for X waves just inside the E-region.

The approximate daytime expressions were derived under the assumption that the conductivity profile in the neighborhood of h_1 could be approximated by a local exponential over an

altitude range of a scale height or so. The approximate nighttime expression was derived under the assumption of a sharp transition in the ionospheric conductivity profile below which the local wavelength is large compared with the local scale height and above which it is small compared with the local scale height. There is an important class of ionospheric-conductivity profiles which bridge the gap between the assumed daytime conditions and nighttime conditions. These are conditions where the altitude h_1 is attained in a region of rapid, but not discontinuous, increase of conductivity with altitude. Under such circumstances, the conductivity profile in the neiborhood of the altitude h_1 cannot be accurately represented by a local exponential. In the following section, the approximate theory of Papers I and II will be extended to deal with such a situation.

III. GENERALIZED CONDUCTIVITY PROFILE

In this section, we consider a more general ionospheric-conductivity profile in the neighborhood of h_1 than those treated in Papers I and II. As in the previous work, h_1 is defined as the altitude at which

$$4\mu_0\omega\sigma_1\zeta_1^2 = 1 \tag{7}$$

where σ_1 is the conductivity at the altitude h_1 and

$$\zeta_1 = \left[\frac{1}{\sigma}\frac{d\sigma}{dz}\right]^{-1}_{z=h_1} \tag{8}$$

is the local scale height at this altitude. A more general conductivity profile in the vicinity of h_1 can be written as

$$\sigma = \sigma_1 \frac{(1+s)\ e^{(z-h_1)/\zeta_2}}{1+se^{(z-h_1)/\zeta_2}} \tag{9}$$

where s is an additional parameter. The scale height ζ_2 is related to the other parameters through

$$\zeta_2 = \frac{\zeta_1}{(1+s)} \tag{10}$$

which follows from Eqs. (8) and (9).

The relationship of Eq. (9) to the conductivity profiles considered in Papers I and II is best exhibited by writing the parameter s as

$$s = e^{(h_1 - h_E)/\zeta_2} \tag{11}$$

where Eq. (11) defines the altitude h_E. The profile given by Eq. (9) then takes the form

$$\sigma = \sigma_1 \frac{(1 + s)\ e^{(z-h_1)/\zeta_2}}{1 + e^{(z-h_E)/\zeta_2}} \ . \tag{12}$$

This profile is approximately exponential up to altitudes a scale height or so below h_E and becomes essentially constant a scale height or so above h_E. This is the kind of transition which typically occurs between the D-region and the E-region, and h_E may thus be thought of as the lower boundary of the E-region. If $h_E \gg h_1$, the profile in the neighborhood of h_1 is a simple exponential with scale height $\zeta_2 \approx \zeta_1$, which was the daytime profile treated in Papers I and II. If $h_E \ll h_1$, the E-region is encountered before the altitude h_1 is attained. If, in addition, the scale height ζ_2 is sufficiently small, the transition between the D- and E-regions is very sharp, which was the nighttime profile treated in Paper II. The generalized conductivity profile given by Eq. (9) thus contains the previous profiles as limiting cases.

IV. DETERMINATION OF APPROXIMATE TEM EIGENVALUES

As in Paper II, we assume that the Earth's magnetic field is vertical. In the QL approximation, propagation in the vicinity of h_1 is governed by two uncoupled wave equations. In terms of quantities $\psi_\pm$ defined in Paper II, these can be written as

$$\frac{\partial^2 \psi_\pm}{\partial z^2} + n_\pm^2 k_0^2 \psi_\pm = 0$$

$$\left(k_0 = \frac{\omega}{c}\right) \tag{13}$$

where

$$n_\pm^2 = \frac{1}{\omega\epsilon_0} (i\sigma_P \pm \sigma_H) . \tag{14}$$

The upper sign corresponds to vertical O wave propagation and the lower sign to vertical X wave (whistler) propagation. Under normal daytime conditions at the upper end of the ELF band and for sufficiently disturbed daytime or nighttime conditions at all ELF frequencies, the altitude h_1 is attained in a region where the ionosphere is nearly isotropic, so that $\sigma_H \approx 0$, $\sigma_P \approx \sigma_0$ and

$$n_+^2 \cong n_-^2 = \frac{i\sigma_0}{\omega\epsilon_0} . \tag{15}$$

Under normal daytime conditions at low frequencies and normal nighttime conditions at all ELF frequencies, h_1 occurs where $|\sigma_H| > |\sigma_P|$, so that

$$n_+^2 \cong -n_-^2 \cong -\left|\frac{\sigma_H}{\epsilon_0 \omega}\right| . \tag{16}$$

It will be assumed that, whichever case applies, the relevant conductivity profile (σ_0 or σ_H) is given by Eq. (9). Both cases may be treated within the same framework by writing

$$n_\pm^2 = e^{i\theta_\pm} \frac{\sigma_1}{\epsilon_0 \omega} \frac{(1+s)\, e^{(z-h_1)/\zeta_2}}{\left[1 + s e^{(z-h_1)/\zeta_2}\right]} \tag{17}$$

where

$$\theta_+ = \theta_- = \frac{\pi}{2} \tag{18}$$

for the isotropic case and

$$\theta_+ = \pi$$

$$\theta_- = 0 \tag{19}$$

for the anisotropic case.

The wave equation may be solved by introducing the variable

$$y = -s e^{(z-h_1)/\zeta_2}$$

which, with Eqs. (7) and (10), transforms Eq. (13) to

$$\frac{d^2 E_\pm}{dy^2} + \frac{1}{y}\frac{dE_\pm}{dy} + \frac{e^{i\theta_\pm}}{4s(1+s)} \frac{1}{y(y-1)} E_\pm = 0. \tag{20}$$

This can be recognized as the generalized hypergeometric differential equation. The solutions which obey the radiation condition at large altitude ($y \to -\infty$) are (Magnus, Oberhettinger, and Soni [1966])

$$\Psi_{\pm} \sim \left(-y\right)^{-\beta_{\pm}} F\left(\beta_{\pm}, \beta_{\pm};\ 1 + 2\beta_{\pm};\ \frac{1}{y}\right) \tag{21}$$

where F is the generalized hypergeometric function, and

$$\beta_{\pm} = \frac{e^{i(\theta_{\pm} - \pi)/2}}{2[s(1+s)]^{1/2}} \ . \tag{22}$$

These solutions are valid down to an altitude a few scale heights below h_1 or h_E, whichever is smaller. At these altitudes ($y \to 0$), the solutions take the form (Magnus, Oberhettinger, and Soni [1966])

$$\Psi_{\pm} \to C_{\pm}[z - G(\beta_{\pm})] \tag{23}$$

where $C_{\pm}$ are constants, and

$$G(\beta) = h_1 - H_2\left[2\psi(1+\beta) - 2\psi(1) - \frac{1}{\beta} + \log s\right] \tag{24}$$

where ψ is the logarithmic derivative of the gamma function.

The eigenvalue is determined by matching Eq. (23) to solutions which obey the proper boundary conditions at the ground and are valid up to altitudes a few scale heights above h_0. These solutions were shown in Paper II to have the form

$$\psi_{\pm} = A\left[(1 \pm C)\, z - S_o^2 \int_o^{z} \frac{dz}{\left(1 + \dfrac{i\sigma_o}{\varepsilon_o \omega}\right)}\right] \tag{25}$$

where A and C are constants. For $z \gg h_0$, the integrand in Eq. (25) becomes very small, and little error is made by extending the range of integration to infinity. Matching Eqs. (23) and (25) in the region where both are valid then leads to the condition

$$S_o^2 = \frac{2G(\beta_+)\, G(\beta_-)}{G(\beta_+) + G(\beta_-)} \; \frac{1}{\displaystyle\int_o^{\infty} \frac{dz}{\left(1 + \dfrac{i\sigma_o}{\varepsilon_o \omega}\right)}} \tag{26}$$

which is the generalized expression for the TEM eigenvalue.

V. COMPARISON WITH PREVIOUS RESULTS

The approximate expressions derived in Papers I and II can be recovered as limiting cases of Eq. (26). In those papers, it was assumed that the conductivity profile in the neighborhood of h_0 could be represented by a local exponential, i.e.,

$$\sigma_o = \epsilon_0\omega e^{(z-h_0)/\zeta_0} . \qquad (27)$$

With this approximation, the integral in Eq. (26) can be evaluated *analytically*, giving

$$\int_o^{\infty} \frac{dz}{\left(1 + \frac{i\sigma_o}{\epsilon_o\omega}\right)} = h_0 - \frac{i\pi}{2}\zeta_0 . \qquad (28)$$

The eigenvalue can thus be written as

$$S_o^2 = \frac{f(\gamma_+,\gamma_-)}{\left(h_0 - \frac{i\pi}{2}\zeta_0\right)} \qquad (29)$$

where

$$f(\gamma_+,\gamma_-) = \frac{2G(\gamma_+)\,G(\gamma_-)}{G(\gamma_+) + G(\gamma_-)} . \qquad (30)$$

The limiting forms of $f(\gamma_+,\gamma_-)$ will now be determined.

a. <u>Daytime ionospheric conditions</u>--In the previous work, it was assumed that, for daytime conditions, the altitude h_i

was below the bottom of the E-region and that the conductivity profile in the neighborhood of h_1 could be approximated by an exponential. As discussed in Section III, this is the limit $h_E \gg h_1$, $s \ll 1$ of the more general conductivity profile. In this limit,

$$\nu_1 \approx \frac{e^{i(\phi_1-\pi)/2}}{2s^{1/2}} . \qquad (31)$$

Since $|\nu_1| \gg 1$, we may use the asymptotic expansion of the ψ function to obtain

$$\psi(1+\nu_1) \approx \log \nu_1 = \frac{i(\phi_1-\pi)}{2} - \log 2 - \frac{1}{2}\log s. \qquad (32)$$

Combined with Eq. (24), this gives

$$G(\phi_1) = h_1 - \ell_2[i(\phi_1 - \pi) - 2(\gamma - \log 2)] \qquad (33)$$

where γ is Euler's constant. For the isotropic daytime ionosphere of Paper I, $\phi_\pm = \frac{\pi}{2}$, so that

$$f(\phi_+, \phi_-) = h_1 + \ell_2\left[\frac{i\pi}{2} + 2(\gamma - \log 2)\right]. \qquad (34)$$

In the limit $s \ll 1$, $\ell_2 \approx \ell_1$. Thus, apart from a small correction term, which was neglected in Paper I, the eigenvalue obtained from Eqs. (29) and (34), agrees with Eq. (1).

For the anisotropic daytime ionosphere of Paper II, $\phi_+ = \pi$ and $\phi_- = 0$. Thus,

$$f(\beta_+, \beta_-) \approx \frac{h_1 (h_1 + i\pi\zeta_2)}{h_1 + \frac{i\pi}{2}\zeta_2} \tag{35}$$

where the small correction term has been neglected. Since $\zeta_2 \approx \zeta_1$, this gives the same eigenvalue as Eq. (4).

b. <u>Nighttime ionospheric conditions</u>--It was assumed in Paper II that, for nighttime conditions, a sharp E-region bottom was encountered before the altitude h_1 was attained, and that the conductivity increased slowly above the E-region bottom. As discussed in Section III, this is the limit $h_E << h_1$, $s >> 1$ of the more general profile. To examine this limit, it is convenient to write

$$\beta_\pm = e^{i(\theta_\pm - \pi)/2} \left(\mu_o \omega \sigma_\infty \zeta_2^2\right)^{1/2} \tag{36}$$

which follows from Eqs. (7), (9), and (10) with

$$\sigma_\infty = \sigma(z \to \infty) = \sigma_1 \frac{(1 + s)}{s} \approx \sigma_1 . \tag{37}$$

In the limit $s >> 1$, $|\beta_\pm| << 1$ and $\Gamma(1 + \beta) \approx \Gamma(1)$. Furthermore,

$$\log s = (h_1 - h_E)/\zeta_2 . \tag{38}$$

It then follows from Eqs. (24) and (30) that

$$f(\beta_+, \beta_-) \approx h_E \frac{(1 + \text{[illegible]})(1 + i\,\text{[illegible]})}{1 + \frac{\text{[illegible]}}{2}(1 + i)}$$

$$\approx h_E \left[1 + \frac{\text{[illegible]}}{2}(1 + i)\right] \tag{39}$$

where

$$\varepsilon = \frac{1}{k_o(\mu_o\sigma_\infty\omega)^{1/2}h_E} = \frac{1}{k_o n_E h_E} . \qquad (40)$$

To lowest order in ε, the eigenvalue obtained from Eqs. (29) and (39) agrees with Eq. (5).

We have thus demonstrated that the results of Papers I and II can be recovered from the more general ionospheric conductivity profile.

VI. APPROXIMATE EXPRESSIONS FOR EXCITATION FACTOR

The ELF signal strength at a receiver depends not only on the propagation loss between transmitter and receiver, but also on the excitation factor. Predictive calculations of the effects of ionospheric disturbances on ELF signal strengths must therefore take this factor into account. In Papers I and II, approximate expressions were developed only for the phase speed and attenuation rate. In this section, the theory will be augmented with approximate analytic expressions for the excitation factor as well.

For the calculation of the excitation factor, it is convenient to reformulate the theory somewhat. Following a customary procedure in ELF calculations, we first make a Fourier decomposition in the transverse plane by writing

$$\vec{B}(\rho,\phi,z) = \frac{k_o^2}{(2\pi)^2} \iint \hat{\vec{B}}(\vec{S},z)\, e^{ik_o\vec{S}\cdot\vec{\rho}}\, d\vec{S} \tag{41a}$$

$$\vec{E}(\rho,\phi,z) = \frac{k_o^2}{(2\pi)^2} \iint \hat{\vec{E}}(\vec{S},z)\, e^{ik_o\vec{S}\cdot\vec{\rho}}\, d\vec{S} \tag{41b}$$

where the hat is used to denote the Fourier transform. We next introduce a rectangular set of $\vec{S}$-dependent unit vectors

$$\vec{e}_1 = \frac{\vec{S}}{S}$$

$$\vec{e}_2 = \vec{e}_3 \times \vec{e}_1$$

$$\vec{e}_3 = \vec{e}_z \tag{42}$$

and resolve all Fourier transforms into their $\dot{S}$-dependent components. When this is carried out, Maxwell's equations separate, in the isotropic lower ionosphere, into two sets of uncoupled equations, one governing the propagation of TE fields and the other of TM fields. The equations for the TM fields, which include the TEM mode, are

$$ic\hat{B}_2 = \frac{1}{\left(1 - \frac{S^2}{n^2}\right)} \frac{\partial \hat{E}_1}{\partial \bar{z}} \tag{43a}$$

$$ic \frac{\partial \hat{B}_2}{\partial \bar{z}} = - n^2 \hat{E}_1 \tag{43b}$$

where $\bar{z} = k_o z$ and

$$n^2 = 1 + \frac{i\sigma_o}{\varepsilon_o \omega} . \tag{44}$$

If we introduce as a variable the wave impedance

$$Z = \frac{\hat{E}_1}{ic\hat{B}_2} , \tag{45}$$

the TM field equations can be combined into the single equation

$$\frac{\partial Z}{\partial \bar{z}} = 1 - \frac{S^2}{n^2} + n^2 Z^2 \tag{46}$$

which is a nonlinear, first-order differential equation of the Riccati type.

We consider a small horizontal electric dipole of strength IL, oriented along the x-direction ($\phi=0$), and located at a height z_s in a thin uniform "source layer" above the ground. The solution in the source layer can be written as

$$E_1(\bar{z}) = - \frac{\mu_o ILcS_x C}{2S(1+rR)} \left[e^{iC\bar{z}_>} + Re^{-iC\bar{z}_>} \right] \left[e^{-iC\bar{z}_<} - re^{iC\bar{z}_<} \right] \tag{47}$$

where $\bar{z}_>$ is the greater of $\bar{z}$ and $\bar{z}_s$ and $\bar{z}_<$ is the smaller of these two quantities, and

$$C = (1 - S^2)^{1/2}. \tag{48}$$

The quantities R and r are reflection coefficients to be determined from the boundary conditions at the upper and lower boundaries of the source layer, respectively. These quantities can be expressed in terms of the wave impedances at these boundaries by the relations

$$r = \frac{iC - Z_L(0)}{iC + Z_L(0)} \tag{49a}$$

$$R = \frac{Z_U(0) + iC}{Z_U(0) - iC} \tag{49b}$$

where we have placed the source layer at the ground ($z=0$) and allowed it to become infinitesimally thin.

The boundary conditions require the continuity of the tangential electric and magnetic fields at all interfaces.

This allows us to evaluate $Z_L(0)$ from the solutions in the ground. In deriving the approximate eigenvalues, it was assumed that the ground was perfectly conducting. The finite conductivity of the ground introduces only a small correction to these expressions. However, the assumption of infinite ground conductivity cannot be made in deriving the excitation factor for a horizontal electric dipole at the ground, since there would be no excitation of the waveguide at all under those conditions. The solutions in the ground obeying the radiation condition have the form

$$E_1, B_2 \propto e^{-i\bar{q}_G \bar{z}} \qquad (59a)$$

where

$$\bar{q}_G = \left(n_G^2 - S^2\right)^{1/2} \approx n_G . \qquad (59b)$$

It then follows from Eqs. (43b) and (45) that

$$Z_L(0) = \frac{i}{n_G} . \qquad ([illegible])$$

The impedance $Z_U(0)$ is determined from the solutions above the source. The wave equation for the impedance can be written as the integral equation

$$Z = Z_U(0) + \int_0^{\bar{z}} \left[1 - \frac{S^2}{n^2} + n^2 Z^2\right] d\bar{z} . \qquad ([illegible])$$

It is not difficult to show that, for $\bar{z} \ll \bar{h}_1$, the contribution to the integral from the last term in the square brackets is small. Thus,

$$Z \cong Z_U(0) + \bar{z} - S^2 \int_0^{\bar{z}} \frac{d\bar{z}}{n^2} \qquad (53)$$

where n^2 is given by Eq. (44). Furthermore, if $z \gg h_0$, little error is made in extending the range of integration to infinity. The wave impedance above the source thus takes the form

$$Z \cong Z_U(0) + k_0 z + k_0 \check{h}_0$$

$$(h_0 \ll z \ll h_1) \qquad (54)$$

where $\check{h}_0$ is defined by the equation

$$\check{h}_0 = \int_0^{\infty} \frac{dz}{\left[1 + \frac{i\sigma}{\varepsilon_0 \omega}\right]} \,. \qquad (55)$$

For an exponential conductivity profile, $\check{h}_0$ is given by the right-hand side of Eq. (28).

From the solutions given in Papers I and II for the region near h_1, the impedance somewhat below h_1 takes the form

$$Z \cong \text{const.}(z - h_1)$$

$$(z \approx h_1) \qquad (56)$$

where

$$\tilde{h}_1 = f(\varepsilon_+, \varepsilon_-) \tag{57}$$

and $f(\varepsilon_+, \varepsilon_-)$ is given by Eq. (30). For the particular profiles treated in previous work, $f(\varepsilon_+, \varepsilon_-)$ reduces to Eqs. (34), (35), or (39), as the case may be.

By requiring the leading terms of Eqs. (54) and (56) to agree in the region $h_0 \ll z \ll \tilde{h}_1$ where both are valid, we obtain

$$Z_U(0) = k_0(S^2 h_0 - \tilde{h}_1). \tag{58}$$

It should be noted that at this point, we may regard S^2 as an undetermined parameter, rather than the TEM eigenvalue.

With $Z_L(0)$ and $Z_U(0)$ now determined, we can express the reflection coefficients r and R in terms of the parameter S by means of Eqs. (49a) and (49b). This, finally, allows us to express E_1 and B_2 in terms of this parameter. The expression for $\tilde{B}_2$ at the ground becomes

$$B_2(0) \cong - \frac{\mu_0 I L S_x}{n_G S k_0 \left(S^2 h_0 - \tilde{h}_1\right)} \tag{59}$$

where we have neglected terms of relative order n_G^{-1}, which are small at ELF.

It now remains to carry out the indicated integration in Eq. (41a). Carrying out the integration over angle in S-space, we obtain

$$\vec{B}(\rho,\phi,0) = -\frac{\mu_0 IL}{2\pi n_G}(\dot{e}_z \times \hat{\rho})\cos\phi \int_0^{\infty} \frac{J_1(k_0 S\rho)}{(S^2\tilde{h}_0 - \tilde{h}_1)}\, dS. \qquad (60)$$

In the remaining integration, the integral can be extended to $-\infty$ by replacing the Bessel function by an appropriate Hankel function. The integral can then be evaluated by contour integration in the complex S-plane. At large distances from the source, the primary contribution is from the pole of the integrand at

$$S_0^2 = \frac{\tilde{h}_1}{\tilde{h}_0} \qquad (61)$$

which will be recognized as the approximate TEM eigenvalue obtained previously. This may be considered as an alternative derivation of that expression. From the residue at this pole, we obtain

$$\vec{B} \sim -\frac{\mu_0 IL\cos\phi}{2n_G(\lambda_0\rho S_0)^{1/2}\tilde{h}_0}\, e^{ik_0 S_0\rho}\, \dot{e}_\phi \qquad (62)$$

where λ_0 is the wavelength in free space and we have used the large argument expansion of the Hankel function.

We define the magnetic excitation factor Λ_B by the equation

$$B_\phi = \Lambda_B \frac{\mu_0 IL}{2n_G(\lambda_0\rho)^{1/2}}\cos\phi\, e^{ik_0 S_0\rho} \qquad (63)$$

which differs somewhat from other definitions (e.g., Bannister [1975]) in separating the effect of the ionosphere from that of the ground. With this definition, the excitation factor becomes

$$\Lambda_B = \tilde{h}_0^{-1} S_0^{-1/2} = \tilde{h}_0^{-3/4} \tilde{h}_1^{-1/4} \tag{64}$$

where h_1 is given by Eq. (57). Finally, since $E_z \; S_0 B_y$, the analogous excitation factor for the vertical electric field is

$$\Lambda_E = \tilde{h}_0^{-5/4} \tilde{h}_1^{1/4}. \tag{65}$$

It was pointed out in Paper II that the horizontal rate of energy flow is essentially constant up to an altitude h_o, above which it falls off very rapidly with altitude. It is therefore not surprising that the effective waveguide height for excitation is roughly h_o, rather than the higher reflecting height h_1.

VII. CONCLUSIONS

The approximate theory developed in Papers I and II for the determination of ELF propagation constants in the earth-ionosphere waveguide has been extended to a rather more general ionospheric-conductivity profile than those considered previously. The results of Papers I and II are recovered as limiting cases of the more general profile. Simple analytic expressions have also been derived for the excitation factor, which is another important parameter in ELF propagation. Although approximate excitation factors have not been compared with full-wave calculations, it is reasonable to assume, on the basis of the generally excellent agreement between approximate and full-wave propagation constants, that they represent very good approximations. The simple analytic expressions derived in this work allow the rapid computation of phase speeds, attenuation rates, and excitation factors for a wide range of ionospheric conditions without the necessity of lengthy full-wave numerical calculations. The theory can therefore be quite useful in the study of the effects of a variety of ionospheric disturbances, both natural and artificial, on ELF communication systems.

BIBLIOGRAPHY

Bannister, P. R., "Variations in Extremely Low Frequency Propagation Parameters," *J. Atmos. Terr. Phys.*, Vol. 37, pp. 1203-1210, 1975.

Bannister, P. R., "Some Notes on ELF Earth-Ionosphere Waveguide Daytime Propagation Parameters," *IEEE Trans. on Antennas and Propagation*, AP-27, 5, pp. 696-698, Sept. 1979.

Greifinger, C., Greifinger, P. "Approximate Method for Determining ELF Eigenvalues in the Earth-Ionosphere Waveguide," *Radio Science*, Vol. 13, pp. 831-837, 1978.

Greifinger, C., and Greifinger, P., "On the Ionospheric Parameters which Govern High Latitude ELF Propagation in the Earth-Ionosphere Waveguide," *Radio Science*, 1979 (to be published).

Magnus, W., Oberhettinger, F., and Soni, R. P., *Formulas and Theorems for the Special Functions of Mathematical Physics*, Springer-Verlag, New York, pp. 48-57, 1966.

Wait, J. R., and Spies, K. P., "Characteristics of the Earth-Ionosphere Waveguide for VLF Radio Waves," National Bureau of Standards, Technical Note 300, 1964.

Wait, J. R., *Electromagnetic Waves in Stratified Media*, Pergammon Press, New York, 1970.

DISTRIBUTION LIST

DEPARTMENT OF THE AIR FORCE (Continued)

Air Weather Service, MAC
Department of the Air Force
ATTN: DNXP, R. Babcock

Assistant Chief of Staff
Studies & Analyses
Department of the Air Force
ATTN: AF/SASC, C. Rightmeyer

Ballistic Missile Office
Air Force Systems Command
ATTN: MNNXH, J. Allen

Deputy Chief of Staff
Research, Development, & Acq
Department of the Air Force
ATTN: AFRDS

Headquarters Space Division
Air Force Systems Command
ATTN: SKY, C. Kennedy

Headquarters Space Division
Air Force Systems Command
ATTN: [illegible], W. Mercer

Headquarters Space Division
Air Force Systems Command
ATTN: E. Butt

Strategic Air Command
Department of the Air Force
ATTN: [illegible]
ATTN: NRT

OTHER GOVERNMENT AGENCIES

Central Intelligence Agency
ATTN: OSWR/NED

Institute for Telecommunications Sciences
ATTN: W. Utlaut

DEPARTMENT OF ENERGY CONTRACTORS

EG&G, Inc
ATTN: D. Wright
ATTN: [illegible] lvin

Lawrence Livermore National Lab
ATTN: L-389, R. Ott
ATTN: L-31, R. Hager

Los Alamos National Scientific Lab
ATTN: D. Simons
ATTN: E. Jones
ATTN: MS 664, J. Zinn
ATTN: MS 670, J. Hopkins

Sandia National Lab
ATTN: Org 4231, T. Wright
ATTN: [illegible]

[illegible]

[illegible]

DEPARTMENT OF DEFENSE CONTRACTORS (Continued)

Aerospace Corp
ATTN: C. Olsen
ATTN: I. Strauss
ATTN: V. Josephson
ATTN: N. Stockwell

University of California at San Diego
ATTN: H. Booker

Charles Stark Draper Lab, Inc
ATTN: D. Cox
ATTN: J. Gilmore

Communications Satellite Corp
ATTN: G. Fang

Cornell University
ATTN: M. Kelly

ESL, Inc
ATTN: J. Marshall

General Electric Co
ATTN: M. Bortner
ATTN: A. Harcar

General Research Corp
ATTN: J. Ise, Jr
ATTN: J. Garbarino

Horizons Technology, Inc
ATTN: R. Kruger

HSS, Inc
ATTN: D. Hansen

Institute for Defense Analyses
ATTN: E. Bauer

JAYCOR
ATTN: J. Sperling

Johns Hopkins University
ATTN: T. Potemra

Kaman TEMPO
ATTN: W. Knapp
ATTN: DASIAC
ATTN: T. Stephens
ATTN: W. McNamara
ATTN: [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

DEPARTMENT OF DEFENSE CONTRACTORS (Continued)

Meteor Communications Consultants
ATTN: R. Leader

Mission Research Corp
ATTN: D. Sappenfield
ATTN: Tech Library
ATTN: F. Fajen
ATTN: R. Bogusch
ATTN: R. Hendrick
ATTN: R. Kilb
ATTN: S. Gutsche
5 cy ATTN: Document Control

Mitre Corporation
ATTN: B. Adams

Mitre Corporation
ATTN: W. Hall
ATTN: W. Foster
ATTN: J. Wheeler

Pacific-Sierra Research Corp
ATTN: H. Brode

Physical Dynamics, Inc
ATTN: E. Fremouw

R & D Associates
ATTN: F. Gilmore
ATTN: W. Karzas
ATTN: R. Lelevier
ATTN: B. Gabbard
ATTN: F. Gerlfinger
ATTN: P. Greifinger
ATTN: C. Hara

R & D Associates
ATTN: [illegible]

DEPARTMENT OF DEFENSE CONTRACTORS (Continued)

Science Applications, Inc
ATTN: [illegible]

SRI International
ATTN: R. [illegible]
ATTN: [illegible]
ATTN: W. Chesnut
ATTN: [illegible]
ATTN: M. Baron
ATTN: [illegible]
ATTN: W. [illegible]

Sylvania Systems Group
ATTN: M. [illegible]

Technology International Corp
ATTN: W. Boquist

TRW Defense & Space Sys Group
ATTN: [illegible]

Utah State University
ATTN: [illegible]

Visidyne, Inc
ATTN: [illegible]
ATTN: [illegible]

[illegible]
ATTN: [illegible]
ATTN: [illegible]

[illegible]
ATTN: [illegible]
ATTN: [illegible]

DATE
ILMED
-8